Aerodynamics and Noise of Coaxial Jets

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The objective of this investigation was to develop a unified prediction method for estimating the aerodynamic and noise characteristics of jets issuing from nozzles of arbitrary geometric shapes. The method has been developed and demonstrated for dual-flow coaxial jets. An extension of Reichardt's theory is utilized to predict the mean velocity, temperature, and axial turbulence intensity distributions throughout the jet plume. The generic noise intensity spectrum is synthesized by a "slice-of-jet" approach, wherein each axial location in the plume contributes to the sound generation in one dominant frequency band. The propagation of the source spectrum to the far field is modeled by means of convected quadrupoles embedded in a parallel slug flow. Extensive predictions of the aeroacoustic characteristics of coaxial jets were made and compared with experiment. The agreement between theory and experiment is quite good, except at high frequencies and shallow angles to the jet axis, where refraction is overestimated. A major conclusion drawn from these results is that the noise reduction attained by a coaxial jet comes primarily from a reduction in turbulence intensity.

Nomenclature

A_j	= jet nozzle cross-sectional area
a	= jet nozzle inner stream exit radius
a_{ij}	= azimuthal average of (i, j) quadrupole amplitude
b	= jet nozzle outer stream exit radius
b_m	= mixing layer momentum thickness
b_h	= mixing layer enthalpy thickness
C_m	= momentum spreading rate parameter
C_h	= enthalpy spreading rate parameter
c_1,c_2,c_0	= speed of sound in inner, outer, and ambient regions, respectively
c_p	= specific heat at constant pressure
D	= jet nozzle effective diameter
f	= observed frequency
H	= local stagnation enthalpy relative to ambient
$H_n^{(1)}$	= Hankel function of the first kind, order n
J_n	= Bessel function of the first kind, order n
K_1,K_2,K_0	= propagation constant in inner, outer, and ambient regions, respectively
k_1	= wave number ω/c_0
l	= typical turbulent correlation length scale
M_c	= source convection Mach number U_s/c_0
p	= acoustic pressure
Q_{ij}	= (i, j) quadrupole
R	= distance from nozzle exit to observer
r	= radial coordinate
S	= jet plume cross-sectional area at distance x
St	= source Strouhal number $f(1-M_c \cos\theta)a/U_{1j}$
T	= jet temperature
T_0	= ambient temperature
t	= time
U_1,U_2	= velocity in inner and outer streams
U_s	= source convection velocity
u	= axial (x) component of velocity
v	= radial (r) component of velocity
x	= axial coordinate
Y_n	= Bessel function of the second kind, order n
y,z	= transverse coordinates in x = constant plane
Δ	= Laplacian operator
δ	= delta function
ϕ	= angle between elemental shear stress vector and radial coordinate
α	= angular coordinate of elemental jet
θ	= azimuthal coordinate angle
Θ	= angle between jet axis (x coordinate) and observer
ρ	= fluid density
σ	= radial coordinate of elemental jet
τ	= turbulent Reynolds stress
ω	= radian source frequency
VR	= nozzle exit velocity ratio $(U_2/U_1)_j$
AR	= nozzle exit area ratio $(b^2-a^2)/a^2$
TR	= nozzle exit temperature ratio $(c_2/c_1)_j^2$
SPL	= ⅓-octave band sound pressure level, re: 2×10^{-4} dynes/cm^2
OASPL	= overall sound pressure level, re: 2×10^{-4} dynes/cm^2

Superscripts and Subscripts

$\overline{(\ )}$	= statistical time average
$(\ )'$	= differentiation or fluctuating component
$(\ )^*$	= complex conjugate
$(\tilde{\ })$	= Fourier transform
$(\ )_j$	= value at nozzle exit plane
$(\ )_1$	= value in inner stream or region
$(\ )_2$	= value in outer stream or region
$\langle\ \rangle$	= statistical time average

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Introduction

In the past few years, considerable progress has been made in achieving an understanding of the noise produced by hot and cold round jets. This progress is a direct result of careful and accurate jet noise measurements[1-4] and new theoretical developments.[5-7] The latter have focused on one important aspect of jet noise, namely acoustic/mean-flow interaction.

It was recognized that a successful jet noise prediction method must include the modeling of both noise generation and propagation. The former is intimately tied to turbulence, an area not understood well quantitatively. Experimental turbulence data relevant to jet noise are somewhat limited,[8-12] but it appears that, through various similarity arguments, it is possible to predict the turbulence characteristics of round jets quite well. Similarly, the propagation aspects of round jet noise have been treated satisfactorily. Perhaps the most complete treatment is given by Mani,[6] who found that the jet mean flow has a profound effect on the far-field noise directivity over the entire frequency spectrum. At high frequencies refraction effects become important, as was first demonstrated by Atvars et al.[13] At low frequencies additional (above Lighthill-Ribner theory) convective amplification effects appear.

It was desirable to extend these turbulence and acoustic models to other nozzle configurations. The primary motivation was to develop a tool to study the parametric dependence of noise on nozzle shape. Such a tool would be indispensable in the search for a "quiet" nozzle. A secondary objective was to check the generality of the concepts developed to describe round nozzle jet noise.

In this article, a model of the aeroacoustic characteristics of coplanar, coaxial nozzles is developed. This is the simplest extension of the round jet work. Considerable acoustic data exist for this geometry, and comparisons of predictions with experiment are presented for a wide range of inner-to-outer stream velocity ratios and exhaust area ratios. The measured features of coaxial jet noise are predicted quite well.

Prediction Model: General Remarks

The development of the present prediction method rests on two primary assumptions: 1) the dominant jet noise generation mechanism is the random momentum fluctuations of the small-scale turbulent structure in the mixing regions of the jet plume; and 2) the propagation of this noise to the far-field observer is altered significantly by the surrounding jet flow in which the turbulent eddies are embedded and convecting. Therefore it is proposed that the jet produces an intrinsic noise intensity spectrum directly relatable to the statistical aerodynamic properties of the jet (i.e., mean velocity and density distributions, and local turbulent structure properties such as length-scale, intensity) which is modified by acoustic/mean-flow interactions.

The prediction method follows a sequence of three basic steps: 1) prediction of the aerodynamic characteristics (mean velocity, density, and turbulent structure properties); 2) evaluation of the turbulent mixing noise at 90 deg to the jet axis utilizing the flow properties from 1 and the Lighthill-Ribner theory; and 3) construction of the far-field sound spectrum at various observer positions, utilizing the results of 1 and 2, and accounting for the source convection and acoustic/mean-flow interaction using Lilley's equation.

Jet Plume Aerodynamics

As was discussed in the previous section, an aerodynamic prediction of the jet plume is required to provide the strength of the noise sources. The method selected is an extension of Reichardt's theory,[14] which basically synthesizes the complex flows from nozzles of arbitrary geometry by superposition of a suitable distribution of elemental round jet flows. This approach was first suggested by Alexander et al.[15] and applied directly to suppressor nozzle configurations by Lee et al.[16] and Grose and Kendal.[17]

Reichardt's theory is a semiempirical one, based on extensive experimental observations that the axial momentum flux profiles were bell-shaped or Gaussian in the fully developed similarity region (suitably far downstream) of a jet. From this observation, a hypothesis for the relation between axial and transverse momentum flux was formulated which yields a governing equation for the axial momentum flux. For the far downstream similarity region of a round jet with nozzle area A_j and exit velocity U_j the governing equation and solution are as follows:

$$\frac{\partial}{\partial x}\langle \rho u^2 \rangle = \frac{\lambda(x)}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\langle \rho u^2 \rangle\right) \tag{1}$$

$$\langle \rho u^2 \rangle = \rho_j U_j^2 \frac{A_j}{\pi b_m^2} e^{-(r/b_m)^2} \tag{2}$$

where

$$\lambda(x) = \tfrac{1}{2}\, b_m \mathrm{d}b_m/\mathrm{d}x \tag{3}$$

and $b_m(x)$ is the axial momentum mixing region width, taken to be proportional to the axial distance x from the nozzle exit plane,

$$b_m(x) = C_m x \tag{4}$$

The jet spreading rate C_m becomes a key parameter in the theory, and is determined experimentally. The coordinate system is shown in Fig. 1.

Because Eq. (1) is linear, the summation of elemental solutions, Eq. (2), is also a solution. This unique feature of Reichardt's theory allows the construction of quite complex jet flows with relatively simple mathematics. Although more rigorous (but containing just as much empiricism, albeit in different forms) theories are available for simple (round and plane) jets, there is no other technique available which offers the capability for modeling jet flows typical of aircraft engine suppressor nozzles such as multitube, lobe, and chute nozzles, etc.

Consider a distribution of elemental jets issuing parallel to the x axis, whose exit areas lie in the $x=0$ plane. Each elemental jet has an exit area $A_j = \sigma\, \mathrm{d}\sigma\, \mathrm{d}\alpha$, located at $(\sigma, \alpha, 0)$, as shown in Fig. 1. The axial momentum flux at a downstream point (r, θ, x) due to the elemental jet exhausting at $(\sigma, \alpha, 0)$ is given by Eq. (2)

$$\mathrm{d}\langle \rho u^2 \rangle = \rho_j U_j^2 (\sigma\, \mathrm{d}\sigma\, \mathrm{d}\alpha / \pi b_m^2) e^{-(\xi/b_m)^2} \tag{5}$$

where

$$\xi^2 = r^2 + \sigma^2 - 2r\sigma\cos(\theta - \alpha)$$

Integrating Eq. (5), the following solution is obtained:

$$\langle \rho u^2 \rangle (r,\theta,x) = \frac{1}{\pi b_m^2}\iint (\rho_j U_j^2) e^{-(\xi/b_m)^2}\sigma\, \mathrm{d}\sigma\, \mathrm{d}\alpha \tag{6}$$

From the distribution of $(\rho_j U_j^2)$ in the exit plane, the local value of $\langle \rho u^2 \rangle$ at any point (r, θ, x) can be found from Eq. (6) by standard numerical integration. Assuming that the jet plume stagnation enthalpy flux H diffuses in the same manner as axial momentum, and analogous expression for stagnation enthalpy flux $\langle \rho u H \rangle$ can be derived,

$$\langle \rho u H \rangle (r,\theta,x) = \frac{1}{\pi b_h^2}\iint (\rho_j U_j H_j) e^{-(\xi/b_h)^2}\sigma\, \mathrm{d}\sigma\, \mathrm{d}\alpha \tag{7}$$

where b_h is the thermal shear layer width, taken to be proportional to x

$$b_h = C_h x, \quad C_h = \text{constant} \tag{8}$$

The stagnation enthalpy is defined as $H = c_p T + \tfrac{1}{2}u^2 - c_p T_0$, and the thermal layer spreading rate C_h also must be obtained experimentally. Assuming that the jet mixing occurs at constant static pressure equal to the ambient value, the solutions for $\langle \rho u^2 \rangle$ and $\langle \rho u H \rangle$ given by Eqs. (6) and (7) are sufficient to determine the distributions of mean axial velocity $\bar{u}$ and temperature $\bar{T}$ throughout the jet plume.

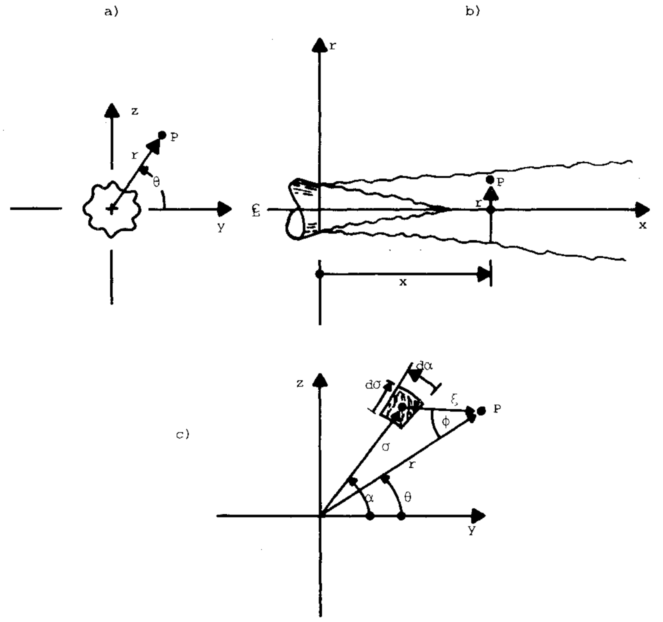

Fig. 1 Jet flow coordinate system and nomenclature: a) nozzle exit plane, b) plume in (r,x) plane, c) nomenclature for elemental jet at (σ,α,0) and field point P.

In addition to the jet plume mean flow properties, the turbulent Reynolds stress, assumed to be proportional to the transverse momentum flux, also can be obtained. Reichardt's hypothesis [from which Eq. (1) evolved] states that the transverse momentum flux is proportional to the transverse gradient of the axial momentum flux, the proportionality factor being $\lambda(x)$. For a simple round jet, from Eqs. (2-4), the Reynolds stress τ is given by

$$\tau = -\langle \rho u' v' \rangle \cong -\lambda \frac{\partial}{\partial r} \langle \rho u^2 \rangle = C_m \rho_j U_j^2 \frac{A_j}{\pi b_m^2} \frac{r}{b_m} e^{-(r/b_m)^2} \tag{9}$$

For an elemental jet exhausting at (σ, α, 0) the shear stress τ at (r, θ, x) lies along a line connecting (σ, α, 0) and the projection of (r, θ, x) onto the $x=0$ plane. This vector is at an angle ϕ to the coordinate direction r (Fig. 1). The radial component of the shear stress $d\tau$ at point (r, θ, x) due to an elemental jet exhausting at (σ, α, 0) is then $d\tau_r = d\tau \cos\phi$. Similarly, the azimuthal component is $d\tau_\theta = d\tau \sin\phi$. Performing the same summation and limiting process over all elemental jets, the total shear stress at (r, θ, x) is then found to be

$$\tau = (\tau_r^2 + \tau_\theta^2)^{1/2} \tag{10}$$

where

$$\tau_r(r,\theta,x) = \frac{C_m}{\pi b_m^2} \iint \rho_j U_j^2 (\xi / b_m) e^{-(\xi/b_m)^2} \cos\phi \; \sigma \, d\sigma \, d\alpha \tag{11}$$

and $\tau_\theta(r,\theta,x)$ is given by a similar expression with $\cos\phi$ replaced by $\sin\phi$. The distance ξ is again given by the expression following Eq. (6), and the angle ϕ is given by

$$\xi \cos\phi = r - \sigma \cos(\theta - \alpha) \tag{12}$$

Equations (5-12) provide the basic expressions for computation of the jet plume flow parameters $\bar{T}$, $\bar{u}$, and τ for a nozzle of arbitrary exit cross section and exit distribution of velocity and temperature.‡ It may be noted that, for axisymmetric nozzles, $\tau = \tau_r$, and $\tau_\theta = 0$. This will be the case for the coaxial jet problem discussed in later sections, but the more general formulation is presented for completeness. The basic limiting assumptions made were: 1) the jet plume mixing occurs at constant static pressure, equal to the ambient value, and 2) the flow is primarily axial with all nozzle exit elements in the same plane $x=0$.

‡The quantities $\langle \rho u^2 \rangle$ and $\langle \rho u H \rangle$ are interpreted as $\bar{\rho}\bar{u}^2$ and $\bar{\rho}\bar{u}\bar{H}$.

Noise Intensity Spectrum at 90 deg

The aerodynamic characteristics of the jet plume provide the information required to evaluate the acoustic intensity spectrum at 90 deg to the jet axis using the Lighthill-Ribner theory of jet noise. This basic spectrum provides a good estimate of the far-field noise spectrum at 90 deg to the jet axis since sound/flow interaction effects are minimal there. Then the noise at any other point in the far field can be computed from this result and the directivity pattern derived in the following section. The analysis presented in the following parallels the work of Lighthill,[18] Ribner,[19] Lilley,[20] and Powell[21]; especially the work of the latter three, as it relates to the use of various similarity arguments.

The far-field mean-square sound pressure, in absence of sound/flow interaction effects is given by[18,19]

$$\langle p^2 \rangle (R,\Theta,\theta) = \frac{x_i x_j x_k x_l}{16\pi^2 c_0^4 R^6} \times \iint \left\langle \frac{\partial^2}{\partial t^2}(\rho v_i v_j) \frac{\partial^2}{\partial t^2}(\rho v_k v_l) \right\rangle d^3y' \, d^3y''$$

where $(\rho v_i v_j)$ is the fluid momentum flux (i,j) component evaluated at vector position y' and time t', and $(\rho v_k v_l)$ is the (k,l) component evaluated at y'' and t''. The retarded times t' and t'' are given by $(t-R'/c_0)$ and $(t-R''/c_0)$, respectively. Note that v_i and x_i denote the i components of velocity and source-to-observer distance R, respectively. Defining separation vectors and time delay $\eta = y' - y''$ and $T = t' - t''$, respectively, and an eddy coordinate $y = \frac{1}{2}(y' + y'')$, the foregoing expression may be written in the following form

$$\langle p^2 \rangle (R,\Theta,\theta) = \frac{x_i x_j x_k x_l}{16\pi^2 c_0^4 R^6} \times \iint \frac{\partial^4}{\partial T^4} \langle (\rho v_i v_j)(\rho v_i v_l) \rangle d^3\eta \, d^3y \tag{13}$$

where $x_1 = R\cos\theta$, $x_2 = R\sin\Theta\cos\theta$, and $x_3 = R\sin\Theta\sin\theta$.

In order to evaluate Eq. (13), some assumptions about the turbulent structure must be made. Because of the lack of detailed experimental or theoretical information approximations are made, following the pioneering work of Ribner,[19] Lilley,[20] and Powell.[21]

The derivatives with respect to T are assumed to be equivalent to multiplying by a typical turbulent eddy fluctuation frequency ω relative to the moving eddy, and the integral over the separation is equivalent to multiplication by l^3, where l is the eddy length scale. Thus,

$$\langle p^2 \rangle (R,\Theta,\theta) \sim \frac{x_i x_j x_k x_l}{16\pi^2 c_0^4 R^6} \int l^3 \omega^4 \langle \rho v_k v_l \rho v_i v_j \rangle d^3y \tag{14}$$

Since the aerodynamic model described in the previous section provides radial and azimuthal distributions of flow properties $\bar{u}$, $\bar{T}$, τ_θ, and τ_r at successive axial stations x, it is convenient to express the volume integration in Eq. (14) as $d^3y = dS(x)\,dx$, where $S(x)$ is the cross-sectional area of the plume. The summation over all components (i, j) of the fluctuating momentum stress tensor in Eq. (14) yields one term that is omnidirectional (self-noise) and another term having a basic directivity of $(\cos^4\Theta + \cos^2\Theta)$. The latter is called shear noise; see, for example, the work of Ribner.[22]

Confining ourselves to $\Theta = 90$ deg for the moment, and referring to $\langle p^2 \rangle$ at $\Theta = 90$ deg as the intrinsic or basic noise spectrum, it is now assumed that $\langle \rho v_i v_j \rangle$ in Eq. (14) is approximately represented by the turbulent shear stress τ. In addition, the typical frequency ω is assumed to be related to $\bar{u}$, ρ, and τ by the equation

$$\omega \sim (\tau/\rho)^{1/2}/l \tag{15a}$$

and that

$$l \sim (x/\bar{u})(\tau/\rho)^{1/2} \tag{15b}$$

These assumed relationships, derived from similarity arguments by Lee et al.,[16] are consistent with the experimental measurements of Davies et al.[8] The basic or intrinsic acoustic pressure level is then as follows, after combining Eqs. (14) and (15):

$$\langle p_0^2 \rangle \sim \frac{1}{16\pi^2 c_0^4 R^2} \iint \rho^2 \bar{u} (\tau/\rho)^{7/2} \mathrm{d}S(x) \frac{\mathrm{d}x}{x} \tag{16}$$

Equations (15) imply that the typical eddy fluctuation frequency at any axial location x is given by

$$f = (\omega/2\pi) \sim (\bar{u}/x) \tag{17}$$

In practice, it was found that, through model calibrations with low-velocity round jet data, Eq. (17) should be modified as follows:

$$fD/\bar{u}_M = 10(x/D)^{-4/3} \tag{18}$$

where $\bar{u}_M$ is taken to be the maximum mean velocity at a given cross section. From Eqs. (16) and (18), the source spectrum can be computed through the approximation, as suggested by Powell,[21]

$$\frac{\mathrm{d}}{\mathrm{d}f}\langle p_0^2 \rangle = \frac{\mathrm{d}x}{\mathrm{d}f}\,\frac{\mathrm{d}}{\mathrm{d}x}\langle p_0^2 \rangle$$

Note that D is a characteristic length scale of the jet nozzle. For a round jet, D is simply the nozzle diameter. For a coaxial jet, D will be defined in a later section. Substituting Eqs. (16) and (18) into the foregoing expression, the following equation for the far-field intrinsic spectrum at $\Theta = 90$ deg results:

$$\mathrm{d}\langle p_0^2 \rangle (f) \sim \frac{\mathrm{d}f/f}{16\pi^2 c_0^4 R^2}\left[\frac{x}{\bar{u}_M}\frac{\mathrm{d}\bar{u}_M}{\mathrm{d}x} - \frac{4}{3}\right]^{-1}$$

$$\times \int \rho^2 \bar{u} (\tau/\rho)^{7/2} \mathrm{d}S(x) \tag{19}$$

To summarize the results of this section, the intrinsic noise spectrum in the absence of convection and refraction effects at $\Theta = 90$ deg is obtained from numerical evaluation of Eq. (19). This expression involves an integration, over the jet plume cross section, of a suitable source strength $[\rho^2 \bar{u} (\tau/\rho)^{7/2}]$, comprised of flow parameters ρ, $\bar{u}$, and τ, which are evaluated from the extended Reichardt model discussed in the previous section. In the following section, the sound/flow interaction modeling approach developed so successfully for simple round jets by Mani[6] will be applied to the coaxial jet problem. This will provide the directional characteristics of the sound sources in the jet flow as a function of flow variables and frequency. By combining the previously generated noise spectrum calculation with the sound/flow interaction prediction model, the net absolute sound spectrum at any observer angle Θ in the far field can then be estimated.

A similar theoretical development was reported by Chen[23] for coaxial jets which employed Reichardt's theory for the aerodynamic prediction and the slice-of-jet concepts[21] for power spectrum prediction. Only power spectrum was considered, and both convection and refraction were ignored. Since power spectrum is the result of integrating sound pressure spectra over a suitably distant sphere in the far field, it can be expected that neglect of sound/flow interaction effects will give incorrect power level estimates at all but the lowest jet velocities.

The Noise Directivity

The previous sections have described how the basic noise intensity is calculated from the turbulence properties of the jet. The noise generated by turbulence in the jet can be modeled as a distribution of convecting sources radiating to the far field at a specific frequency. The objective of this section is to evaluate how a convecting noise source of given strength and frequency radiates to the far field. This radiation pattern differs considerably from the classical result of Lighthill[18] because, in the present formulation, the source is surrounded by a rapidly moving hot jet. The analytical development follows very closely the work of Mani[6]; a detailed discussion of the implications of the simplifying assumptions made herein can be found in that work.

Our starting point is Lilley's equation,[24] which, in the case of a slug flow velocity profile, simplifies to the following equation for acoustic pressure p:

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 p - c^2 \Delta p = \delta(x - U_s t)\delta(r - r_0)\delta(\theta - \theta_0)\frac{e^{-i\omega t}}{r} \tag{20}$$

The jet plume, for the purpose of evaluating the acoustic propagation effects, is modeled as two coaxial slug flow jets, as shown in Fig. 2. A convecting source is located within slug flow at $r_0 < a$ and θ_0. The source moves at velocity U_s in the downstream direction while emitting sound at frequency ω in its own (convecting) reference frame. The slug flow velocity U and speed of sound c take on values of U_1, U_2, U_0, and c_1, c_2, c_0 in each of the three regions (inner flow, outer flow, and ambient), respectively. Note that U_1 is some representative average value of inner stream velocity, not necessarily equal to nozzle exit value; the same remark holds for U_2 in the outer stream.

The solution to Eq. (20), satisfying suitable jump conditions across the fluid interfaces at $r = a$ and $r = b$, and obeying the Sommerfeld radiation condition at $r = \infty$, is obtained by Fourier transforms. Define the Fourier transform of p as

$$\tilde{p} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\pi}^{\pi} p e^{in\theta} e^{-i\Omega t} e^{-isx} \mathrm{d}\theta\, \mathrm{d}t\, \mathrm{d}x \tag{21}$$

whose inverse is

$$p = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \tilde{p} e^{-in\theta} e^{i\Omega t} e^{isx} \mathrm{d}\Omega\, \mathrm{d}s \tag{22}$$

and apply the transformation (21) to Eq. (20). After a number of integrations by parts (and ignoring the contributions from upper and lower limits), we find

$$\frac{\mathrm{d}^2\tilde{p}}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\tilde{p}}{\mathrm{d}r} + \left[\frac{(\Omega + Us)^2}{c^2} - s^2 - \frac{n^2}{r^2}\right]\tilde{p}$$

$$= F\frac{\delta(r - r_0)}{r} e^{in\theta_0} \tag{23}$$

where

$$F = -\delta(\Omega + \Omega_1)/c_1^2, \qquad \Omega_1 = \omega + U_s s \tag{24}$$

Here, Ω, s, and n are the Fourier transform variables.

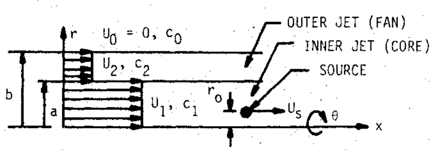

Fig. 2 Geometry of problem.

The solution of Eq. (23) is classical in terms of Bessel functions. The actual form of the solution (whether we use regular or modified Bessel functions) is a strong function of the algebraic sign of

$$K^2 = [(\Omega + Us)^2/c^2] - s^2 \tag{25}$$

We require K^2 in the ambient region, K_0^2, to be positive, since otherwise no propagation takes place in the far field. This places a certain restriction on Ω and s. For these same values of Ω and s, the values of K^2 in regions one (K_1^2) and two (K_2^2) may be positive or negative.

Across the interfaces we must have continuity of pressure and particle displacement. This is because the interface must consist of particles of fixed identity. If $[\![f]\!]$ denotes the jump in f across an interface, we then require that

$$[\![\bar{p}]\!] = 0 \tag{26}$$

$$[\![\frac{1}{\rho} \frac{1}{(\Omega + Us)^2} \frac{\mathrm{d}\bar{p}}{\mathrm{d}r}]\!] = 0 \tag{27}$$

where ρ is the average fluid density in a given region. Since the undisturbed static pressure is assumed to be a constant throughout the jet, ρ is directly calculable in terms of c. Note that for the coaxial jet problem there are two interfaces; one at $r=a$ and another at $r=b$ $(b \geq a)$. Across the source location $r=r_0$, $\bar{p}$ is continuous, and $\mathrm{d}\bar{p}/\mathrm{d}r$ changes by $F \exp(in\theta_0)$.

The foregoing jump conditions and the radiation condition at infinity render the solution to the problem unique. The required solution involves a tremendous amount of algebra involving very lengthy expressions, which need not be reproduced here. The final result for the acoustic pressure in the ambient region is given by

$$p = -\frac{1}{2\pi^2 c_1^2} e^{-i\omega t} \sum_{n=0}^{\infty} \epsilon_n \cos n(\theta - \theta_0)$$

$$\times \int_{-\infty}^{\infty} e^{is(x - U_s t)} \Big[A J_n(K_1 r_0) H_n^{(1)}(K_0 r) \Big]_{\Omega = -\Omega_1} \mathrm{d}s \tag{28a}$$

where ϵ_n is the Neumann factor ($\epsilon_0 = \frac{1}{2}$, $\epsilon_n = 1$, $n \geq 1$), and A is given by

$$A = \frac{W(K_1 a)}{r_0 K_1 W(K_1 r_0)} \left[\frac{\rho_1}{\rho_2} \frac{K_2}{K_1} \right.$$

$$\left. \times \left(\frac{\Omega + U_1 s}{\Omega + U_2 s} \right)^2 \beta J_n(K_1 a) - \alpha J_n'(K_1 a) \right]^{-1} \tag{28b}$$

The parameters α and β in the foregoing expressions for A are given by

$$\alpha = H_n^{(1)}(K_0 b) \frac{W(K_2 a, K_2 b)}{W(K_2 b)}$$

$$+ \frac{\rho_2}{\rho_0} \frac{K_0}{K_2} \frac{(\Omega + U_2 s)^2}{\Omega^2} H_n^{(1)\prime}(K_0 b) \frac{L(K_2 b, K_2 a)}{W(K_2 b)} \tag{28c}$$

$$\beta = H_n^{(1)}(K_0 b) \frac{L'(K_2 a, K_2 b)}{W(K_2 b)}$$

$$+ \frac{\rho_2}{\rho_0} \frac{K_0}{K_2} \frac{(\Omega + U_2 s)^2}{\Omega^2} H_n^{(1)\prime}(K_0 b) \frac{W(K_2 b, K_2 a)}{W(K_2 b)} \tag{28d}$$

The auxiliary functions occurring in Eq. (28) are defined as follows:

$$W(z, \zeta) = J_n(z) Y_n'(\zeta) - J_n'(\zeta) Y_n(z) \tag{29a}$$

$$L(z, \zeta) = J_n(z) Y_n(\zeta) - J_n(\zeta) Y_n(z) \tag{29b}$$

$$L'(z, \zeta) = J_n'(z) Y_n'(\zeta) - J_n'(\zeta) Y_n'(z) \tag{29c}$$

$$W(z) = W(z, z) \tag{29d}$$

where J_n and Y_n are Bessel functions, and the primes denote differentiations. $W(z)$ is, of course, the Jacobian, and $H_n^{(1)}$ is the Hankel function of the first kind.

The preceding solution for the pressure in the ambient field is valid as long as (K_1^2, K_2^2, and K_0^2) >0. When K_1^2 is negative, Eqs. (28) and (29) are still valid provided that all of the Bessel functions are replaced by their modified counterparts. Similar remarks hold for K_2^2. The solution given by Eqs. (27) and (28) represents the acoustic pressure for a simple source convecting with the flow, having a source strength of unity and frequency ω.

The Far Field of Quadrupoles

In principle (say, numerically), it is possible to evaluate the integral in Eq. (28a), and then to differentiate the resultant expression with respect to the source coordinates in order to generate the dipole and quadrupole solutions to Lilley's equation.[6] On the other hand, whenever the observation point is in the far field, it is possible to evaluate the s integral in Eq. (28a) by the method of stationary phase. The technique is classical so we need only quote the final result. In the limit as $(r^2 + x^2)^{1/2} \to \infty$, Eq. (28a) can be reduced to the following:

$$p = \sum_{n=0}^{\infty} B_n \cos n(\theta - \theta_0) J_n(K_1 r_0) \tag{30a}$$

where

$$B_n = \frac{i\epsilon_n}{\pi^2 c_1^2} \frac{e^{-i\omega(t - R/c_0)}}{R(1 - M_c \cos\Theta)} A e^{-in\pi/2} \tag{30b}$$

and R is the distance from the jet nozzle to the observer located at angle Θ with respect to the x axis, and M_c is the source convection Mach number U_s/c_0. Also, A is to be evaluated at the point of stationary phase, given by

$$s = k_1 \frac{\cos\Theta}{1 - M_c \cos\Theta} \tag{30c}$$

where $k_1 = \omega/c_0$. Equations (30) contain the results of Lighthill[18] for the limiting condition $c_1 = c_2 = c_0$, $U_1 = U_2 = 0$, as well as the round jet results of Mani for $a=b$. Thus Eq. (30) is a generalization of previous acoustic theories.

Thus far, the location of the source r_0 has remained arbitrary. Physically, the most appropriate location for the source is along the nozzle lip line (i.e., at $r_0 = a$, and $r_0 = b$). However, in the case of a slug flow model of the fluid shroud, Mani[6] and Balsa[25] have found that the precise location of the source is not too important, and that sources convecting on the jet centerline sufficiently explain most of the characteristics of both hot and cold round jet noise. Thus, in this analysis, we will set $r_0 = 0$.

Equation (30a) is now expanded in a Taylor series about $r_0=0$, yielding the result

$$p=C_0+y_0C_1\cos\theta+z_0C_1\sin\theta+(y_0^2-z_0^2)C_2\cos 2\theta$$

$$+2y_0z_0C_2\sin 2\theta\mp \tfrac{1}{4}K_1^2(y_0^2+z_0^2)C_0+0(r_0^3) \tag{31a}$$

where

$$C_n=\frac{B_n|K_1^2|^{n/2}}{2^n\Gamma(n+1)} \tag{31b}$$

and (y_0, z_0) denote the transverse coordinates of the source. The upper sign in Eq. (31a) is used when $K_1^2>0$, whereas the lower sign is used when $K_1^2<0$, and $\Gamma(n)$ is the Gamma function.

The transverse dipole and quadrupole solutions can be obtained from Eq. (31a) by differentiating with respect to y_0 and z_0 and then setting $r_0=0$. Also, differentiations with respect to x generate longitudinal dipole and quadrupole solutions. This latter operation is equivalent to multiplication by s given by Eq. (30c); symbolically, $\partial/\partial x\rightarrow s$.

As an example, consider the $y-y$ quadrupole $Q_{22}=Q_{yy}$. The solution in terms of the simple source solution is given by Eq. (31a),

$$Q_{22}=Q_{yy}=\left[\frac{\partial^2 p}{\partial y_0^2}\right]_{r_0=0}=2C_2\cos 2\theta\mp \tfrac{1}{2}K_1^2C_0 \tag{32}$$

The square of the amplitude of this quadrupole is given by

$$|Q_{22}|^2=Q_{22}Q_{22}^*$$

where Q^* is the complex conjugate of Q. If we define, for any quadrupole (i,j),

$$a_{ij}=\frac{1}{8\pi^2}\int_0^{2\pi}\int_0^{2\pi}|Q_{ij}|^2\mathrm{d}\theta\,\mathrm{d}\theta_0 \tag{33}$$

we find that

$$a_{22}=C_2C_2^*+\tfrac{1}{8}K_1^4C_0C_0^* \tag{34}$$

Physically, a_{22} is the azimuthal average of the amplitude of a ring of totally incoherent $y-y$ quadrupoles.

The expression for acoustic pressure, Eq. (31a), is valid for a "unit" convecting (and compact) velocity fluctuation. Both in the Lilley and Lighthill formulations, the strength of the noise source is proportional to the jet density. Mani[6] has shown that a compact velocity quadrupole in a heated jet generates dipole and source terms. A detailed derivation of these terms is omitted herein for brevity, but the final expressions are quoted in the following:

$$a_{11}=\frac{1}{2}k_1^4\rho^2\frac{\cos^4\Theta}{(1-M_c\cos\Theta)^4}C_0C_0^* \tag{35a}$$

$$a_{12}=\frac{1}{4}k_1^2\frac{\cos^2\Theta}{(1-M_c\cos\Theta)^2}\left[\left(\frac{\mathrm{d}\rho}{\mathrm{d}r}\right)^2C_0C_0^*+\rho^2C_1C_1^*\right] \tag{35b}$$

$$a_{22}=\frac{3}{16}\left[\left(\frac{\partial^2\rho}{\partial r^2}\right)^2+\frac{1}{r^2}\left(\frac{\partial\rho}{\partial r}\right)^2\right]C_0C_0^*$$

$$+\frac{1}{2}\left(\frac{\partial\rho}{\partial r}\right)^2C_1C_1^*+\rho^2\left[C_2C_2^*+\frac{1}{8}K_1^4C_0C_0^*\right] \tag{35c}$$

$$a_{23}=\frac{1}{16}\left[\left(\frac{\partial^2\rho}{\partial r^2}\right)^2+\frac{1}{r^2}\left(\frac{\partial\rho}{\partial r}\right)^2\right]C_0C_0^*$$

$$+\frac{1}{4}\left(\frac{\partial\rho}{\partial r}\right)^2C_1C_1^*+\rho^2C_2C_2^* \tag{35d}$$

In these equations, $(\rho, \partial\rho/\partial r, \partial^2\rho/\partial r^2)$ are some representative values of the density and its various gradients. The exact computation of these gradients follows the procedures proposed by Mani.[6] Note that, when $\rho=1$, Eq. (35c) reduces to Eq. (34), as required.

Finally, there remains to combine these quadrupoles so that the noise source is effectively an eddy of isotropic turbulence, as suggested by Ribner.[22] In the present terminology, the approximate mean square pressure is given by

$$\overline{p^2}\sim(a_{11}+4a_{12}+2a_{22}+2a_{23}) \tag{36}$$

The factor of proportionality in Eq. (36) is directly relatable to the turbulence properties in the jet supplied by the aerodynamic calculation. If $\overline{p^2}$ is known at one angle (say $\Theta=90$ deg), this factor can be found and Eq. (36) can be used to find the mean square pressure at all other angles. By evaluating Eq. (36) at $\Theta=90$ deg and equating to the expression given by Eq. (19), the constant of proportionality can be evaluated. Thus, the absolute level and directivity in each frequency band can be estimated.

Discussion of Results

In applying the previously described model to coplanar jet noise predictions, three further assumptions had to be made. First, it was found, through examination of flowfield measurements, that the jet momentum spreading parameter C_m varies with outer-to-inner stream velocity ratio as follows:

$$C_m=C_{m_0}/(1+U_{\min}/U_{\max}) \tag{37}$$

where $C_{m_0}\approx 0.075$, and $C_h\approx C_m$, and $U_{\min}$ and $U_{\max}$ are the minimum and maximum values of (U_1, U_2) evaluated at the nozzle exit. This relationship is independent of nozzle area ratio, and reflects the reduction in mixing rate when the streams on both sides of the mixing zone are moving in the same direction. The second assumption made concerns the selection of a diameter of characteristic length D to use in determining the typical frequency of each jet slice, as given by Eq. (18). A suitable expression for D which satisfies the limiting conditions when $U_2=0$ or $U_1=U_2$ is

$$\frac{D}{2}=\frac{U_{1j}}{U_{\max}}a+\frac{U_{2j}}{U_{\max}}(b-a) \tag{38}$$

A final assumption made was that the "suitable average" values of U_1 and U_2 used in evaluation of the directivity

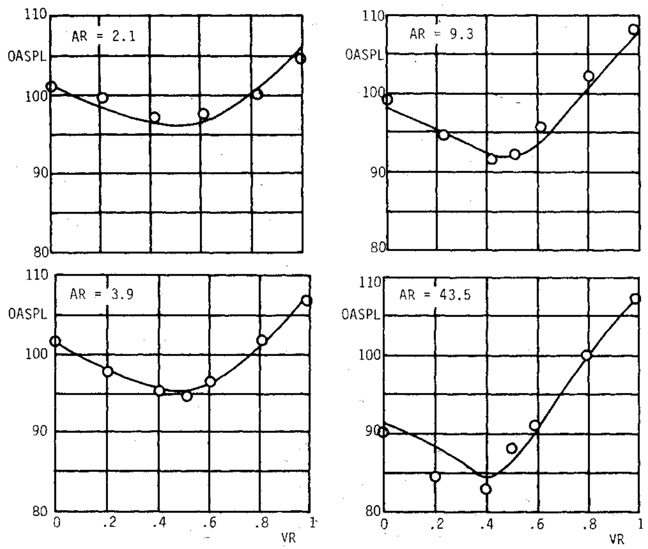

Fig. 3 Overall SPL at $\Theta=90$ deg as a function of area and velocity ratios. Cold jet, $U_{1j}=980$ fps (○ Olsen's data, —— present theory).

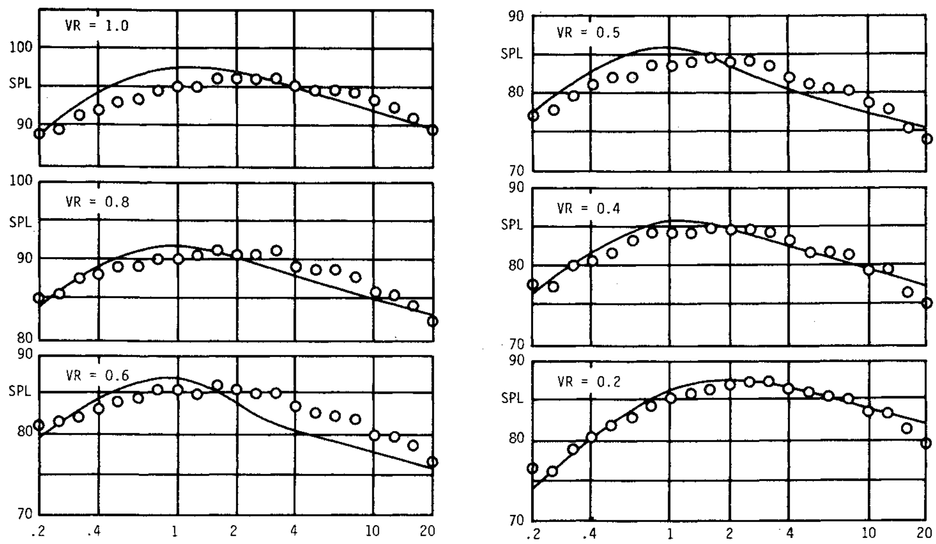

Fig. 4 SPL at $\Theta = 90$ deg as a function of velocity ratio. AR = 3.9. Cold jet, $U_{Ij} = 980$ fps (○ Olsen's data, — present theory).

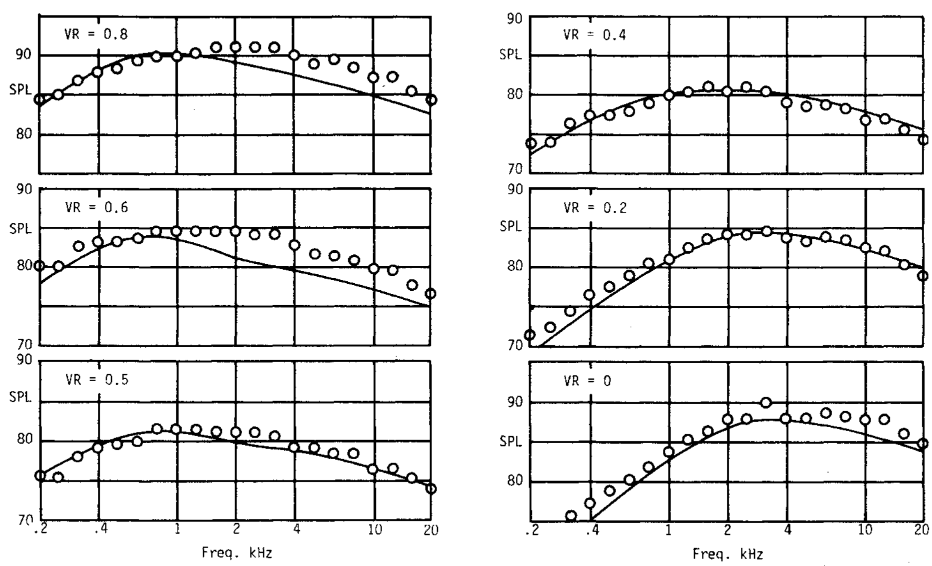

Fig. 5 SPL at $\Theta = 90$ deg as a function of velocity ratio. AR = 9.3. Cold jet, $U_{Ij} = 980$ fps (○ Olsen's data, — present theory).

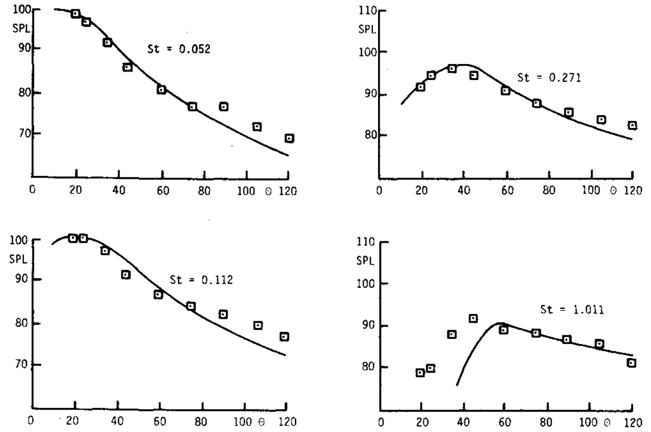

Fig. 6 SPL as a function of source Strouhal number. AR = 3.9, VR = 0.4. Cold jet, $U_{Ij} = 980$ fps (⊡ Olsen's data, — present theory).

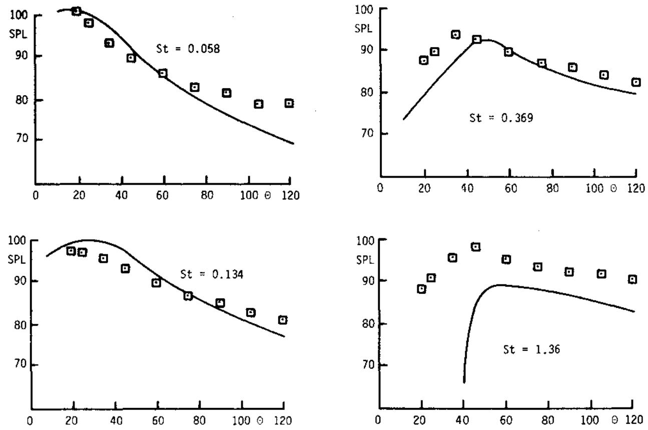

Fig. 7 SPL as a function of source Strouhal number. AR = 3.9, VR = 0.6. Cold jet, $U_{Ij} = 980$ fps (⊡ Olsen's data, — present theory).

expressions of the previous section are given by 65% of the corresponding nozzle exit values. Further, the source convection velocity was assumed to be 65% of U_I evaluated at the nozzle exit.

In Fig. 3 are shown OASPL variations with velocity ratio VR and area ratio AR, at an observer angle $\Theta = 90$ deg. These predictions essentially come from the aerodynamic portion of the prediction model and the Lighthill-Ribner theory of jet noise. The data shown are from Olsen[26] (denoted by symbols), and the theory is indicated by a solid line. There is remarkably good agreement at all area and velocity ratios. In particular, both the location and the magnitude of the noise

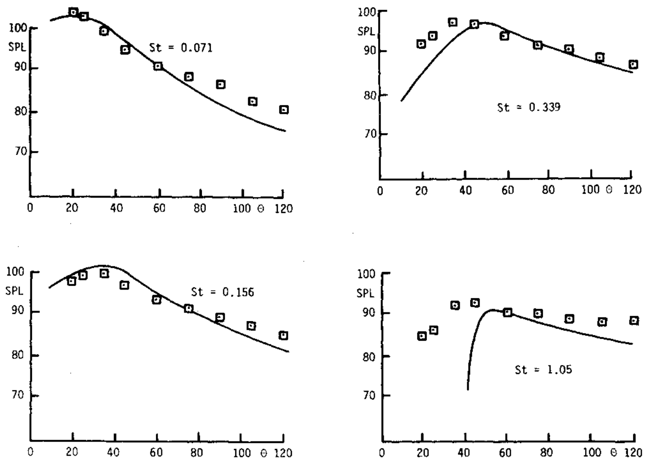

Fig. 8 SPL as a function of source Strouhal number. AR = 3.9, VR = 0.8. Cold jet, $U_{lj} = 980$ fps (□ Olsen's data, — present theory).

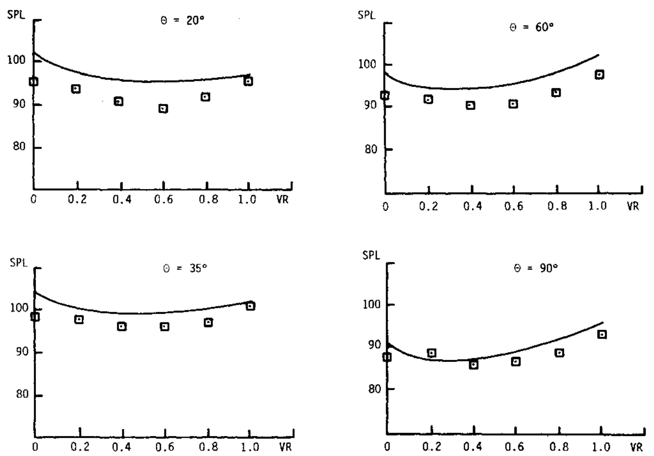

Fig. 9 SPL as a function of velocity ratio. AR = 2.1, $St = 0.3$. Cold jet, $U_{lj} = 980$ fps (□ Olsen's data, — present theory).

minimum is predicted correctly. This noise minimum is a direct consequence of the reduction in turbulence intensity in the inner-to-outer stream mixing layer as the outer flow velocity is increased to about 40% of the inner flow velocity. Further increases in outer flow velocity cause the outer-to-ambient stream mixing layer turbulence to produce the dominant noise. In Figs. 4 and 5 are shown corresponding SPL spectra at two area ratios and several velocity ratios. The agreement attained between theory and experiment was found to be quite good.

In Figs. 6-8 the SPL as a function of observer angle Θ are shown, at constant values of source Strouhal number $St = fa(1 - M_c \cos\Theta)/U_{lj}$. These results, for AR = 3.9, are shown at velocity ratios of 0.4, 0.6, and 0.8, respectively. Again it is recalled that the SPL at Θ = 90 deg comes from the turbulence prediction and the Lighthill-Ribner theory. The acoustic theory, Eqs. (30-36), extends the 90 deg prediction to all other angles. It is seen that the agreement between theory and experiments is good except at high frequencies and shallow angles, where refraction is generally overestimated. This is a limitation of the slug flow assumption, as Mani[6] also obtained similar results for round jets.

In Fig. 9 are shown SPL vs VR trends at several angles and Strouhal numbers. Again, the acoustic theory is quite successful in predicting the directivity pattern, while the basic turbulence/intrinsic intensity models yield the correct absolute levels.

Finally, in Fig. 10, the SPL spectra for a heated coaxial jet are shown at several velocity ratios. Again the agreement between theory and data (from Kazin et al.[27]) is seen to be very good.

Concluding Remarks

In summary, it appears that the present model is capable of predicting many of the observed characteristics, including absolute level, of coaxial jet noise. The noise reduction of coaxial jets, for $VR \leq 1$, was found to be primarily a result of reduction in turbulence intensity. A number of improvements in the theory are currently in progress. These include a better description of the turbulence spectrum (i.e., the slice-of-jet approach is replaced by a local eddy-volume discretization of the jet plume), and the slug flow is replaced by continuous sheared velocity and temperature profiles.

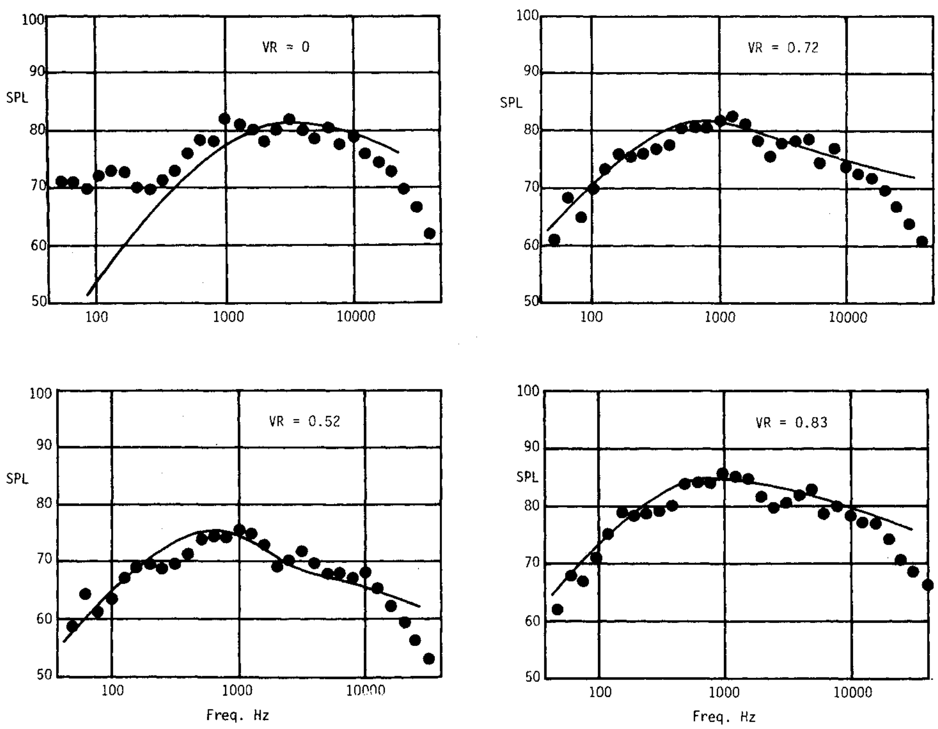

Fig. 10 SPL at Θ = 90 deg as a function of velocity ratio. AR = 4.0, TR = 2.1. Hot jet, $U_{lj} = 1000$ fps, $T_{lj} = 1200°$R (● Kazin's data, — present theory).

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